$$-\tan B = e\,\frac{\tan B+\tan A}{1-\tan B\tan A};$$

$$\tan DAB = \frac{(1+e)\tan B}{\tan^2 B - e}$$

$$= \frac{(1+e)\cot B}{1-e\cot^2 B}.$$

Also solved by *O. W. ANTHONY*, and *J. SCHEFFER*.

PROBLEMS.

35. **Proposed by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Applied Science, Texarkana College, Texarkana, Arkansas-Texas.**

A man weighs 150 pounds; his balloon with all its attachments weighs 500 pounds. What volume of pure hydrogen must be made and put into the balloon so that it will be on the point of ascending with the man? How many kilograms of zinc and of hydrogen sulphate will be used in generating the hydrogen? Give volume of hydrogen in cubic feet given that one litre of hydrogen weighs .0896 grams.

36. **Proposed by O. W. ANTHONY, Professor of Mathematics, New Windsor College, New Windsor, Maryland.**

A vertical slit is made in the middle of the side of a rectangular box containing water. What is the time required to empty the box?

AVERAGE AND PROBABILITY.

Conducted by B. F. FINKEL, Springfield, Mo. All contributions to this department should be sent to him.

SOLUTIONS OF PROBLEMS.

26. **Proposed by J. W. WATSON, Middlecreek, Ohio.**

Find the average area of all right-angled triangles having a *constant* hypotenuse.

III. **Solution by F. P. MATZ, D. Sc., Ph. D., Professor of Mathematics and Astronomy in Irving College, Mechanicsburg, Pennsylvania.**

In my first two solutions I made an arm of the right-angled triangle vary uniformly, although I employed two different systems of co-ordinates. I continued this variation until the arms of the right-angled triangle became equal; and by doing this I avoided all *reciprocal* equal right-angled triangles. Looking at results from this standpoint, the verdict must be—*correct*. By simply varying

uniformly an arm of an inscribed right-angled triangle, as I have done, I am now fully convinced that not all the *possible* right-angled triangles are comprehended; for, most certainly, the uniform variation of an arm does not cause a uniform variation of the vertex of the right angle—along a quadrant of the circumscribing circle. I say the *most natural* solution of this problem is the one in which the *arc* of the circumscribing circle is made to vary uniformly, and this variation is to extend over only one quadrant of the circumscribing circle. Let x=the arc intercepted by an arm and the constant hypotenuse h; then, if $\frac{1}{4}\pi h=a$, the required average area is

$$A=\tfrac{1}{2}h^2\int_0^a \sin\left(\frac{x}{h}\right)\cos\left(\frac{x}{h}\right)dx \div \int_0^a dx=\frac{h^2}{2\pi}\dots\dots(1).$$

FOURTH SOLUTION.

Taking θ as the *central* angle intercepted by an arm, and by the constant hypotenuse, of the right-angled triangle, the required average area becomes

$$A=\tfrac{1}{4}h^2\int_0^{\frac{1}{2}\pi}\sin\theta d\theta \div \int_0^{\frac{1}{2}\pi}d\theta=\frac{h^2}{2\pi}\dots\dots(2).$$

FIFTH SOLUTION.

Making ϕ one of the acute angles, the required average area becomes

$$A=\tfrac{1}{4}h^2\int_0^{\frac{1}{4}\pi}\sin 2\phi d\phi \div \int_0^{\frac{1}{4}\pi}d\phi=\frac{h^2}{2\pi}\dots\dots(3).$$

SIXTH SOLUTION.

Let the origin of Cartesian co-ordinates be placed at the center of the circle; then the required average area becomes

$$A=\tfrac{1}{4}h^2\int_0^{\frac{1}{2}h}dx \div \tfrac{1}{2}h\int_0^{\frac{1}{4}\pi}\frac{dx}{\sqrt{(\frac{1}{4}h^2-x^2)}}=\frac{h^2}{2\pi}\dots\dots(4).$$

Several other solutions leading to the same result are possible.

NOTE.—Professor O. W. Anthony sent us a note in which he defends the solution leading to $\frac{1}{6}a^2$ as the answer. His argument being, in substance, this: The mind does not form a picture of a right triangle inscribed in a semi-circle whose diameter is a but simply a right triangle whose hypotenuse is a. He therefore concludes that the number of triangles should be found by varying one of the sides.

The discussion of this problem called forth the excellent article, "A Note on Mean Values," by Dr. Moore, page 303 of this issue of the MONTHLY. I am quite sure that that article will be greatly appreciated by those of our readers who are interested in this abstruce subject, Mean Value.

Taking the substance of that article as criterion, it remains to determine whether or not the above problem is stated in the *definite* form. I hold the opinion that it is stated in the *definite* form; for the problem requires the average area of *all* right triangles having a given hypotenuse. It does not require the average area of all right triangles having a given hypotenuse and formed according to a certain law, but all the right triangles having a given hypotenuse and the law of formation must be so chosen as to give *all* such right triangles. Therefore, the solutions leading to the result $\frac{a^2}{2\pi}$ are the *correct* and *only* solutions of the problem. EDITOR.]